

# Low Latency Communications

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**Abstract**—Numerous applications demand communication schemes that minimize the transmission delay while achieving a given level of reliability. An extreme case is high-frequency trading whereby saving a fraction of millisecond over a route between Chicago and New York can be a game-changer. While such communications are often carried by fiber, microwave links can reduce transmission delays over large distances due to more direct routes and faster wave propagation. In order to bridge large distances, information is sent over a multihop relay network.

Motivated by these applications, this paper presents an information-theoretic approach to the design of optimal multihop microwave networks that minimizes end-to-end transmission delay. To characterize the delay introduced by coding, we derive error exponents achievable in multihop networks. We formulate and solve an optimization problem that determines optimal selection of amplify-and-forward and decode-and-forward relays. We present the optimal solution for several examples of networks. We prove that in high SNR the optimum transmission scheme is for all relays to perform amplify-and-forward. We then analyze the impact of deploying noisy feedback.

## I. INTRODUCTION

Operating close to the channel capacity requires encoding with large codelengths in order to guarantee diminishing probability of error. In turn, large codelengths introduce decoding delay at the receiver. If data is sent over a multihop network, this delay can multiply over multiple hops, thereby increasing the end-to-end latency. On the other hand, numerous applications, instead of striving to operate at the maximum rate, demand communications with the minimum latency. An extreme case is high-frequency trading in which profits depend on computer-based algorithmic trades that are made as fast as possible. In these settings, in order to bridge large distances, information is sent over a multihop relay network (see Fig. 1). At the same time, saving a fraction of millisecond over, for example, a route between Chicago and New York, can be a game-changer. According to [1], 1 ms of reduced delay translates into \$100 million profit per year. On the Chicago-New York route, fiber can deliver data in 6.6 ms. On the other hand, the latest microwave network can deliver data in 4.1 ms [2]. The gain in the microwave transmission comes from faster wave propagation in the air when compared to the fiber, and from shorter routes. In this paper, we are concerned with such low-latency communications.

End-to-end transmission delay drastically varies with the choice of the cooperative scheme used by a relay. In particular, a relay performing decode-and-forward (DF) will introduce a delay of the order of the size of the block,  $n$ , that it needs

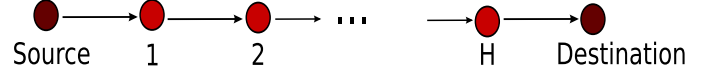


Fig. 1. Multihop network. Relays are labeled as 1,...,H

to receive prior to decoding, re-encoding and forwarding a message. In contrast, an amplify-and-forward (AF) relay can forward on per-symbol basis, thus introducing a roughly  $n$  times smaller delay. However, the simplicity of AF comes at the expense of amplifying and propagating the noise thereby reducing the effective received signal-to-noise ratio with every subsequent AF hop. The reduced SNR reduces the transmission rate and ultimately, after a sequence of AF hops, results in a higher delay when compared to DF.

In microwave low-latency networks, common practice is to perform DF or AF at a node uniquely on the basis of its received SNR. A relay with a received SNR above a certain threshold performs AF, otherwise, it performs DF. Typically, this criterion performs DF at a relay after a several AF hops or after one long hop.

Our goal is to design a multihop microwave network to minimize the end-to-end delay. Towards that goal, we address several practical questions: 1) Is the common practice of assigning AF/DF relays based on received SNR optimal? 2) If not, when should DF relays be used given that they introduce larger delays? 3) Does this selection depend on the SNR regime the network operates in? 4) Can the delay be reduced by deployment of noisy feedback?

In this paper, we answer the above questions. To characterize the delay introduced by coding, we derive error exponents achievable in a multihop network. The error exponent characterizes the tradeoff between the code block size (and hence the delay) and the reliability [3]. Given the desired reliability and using the error exponent, we obtain the lower bound on the delay in the considered multihop network. We then formulate and solve an optimization problem that determines optimal selection of amplify-and-forward and decode-and-forward relays. We demonstrate that an approach in which the selection of AF and DF scheme at a relay is solely based on the received SNR is suboptimal. We present the optimal DF/AF selections for several examples of networks. We prove that in high SNR the optimum transmission scheme is for all relays to perform amplify-and-forward. We show that in a symmetric network, all decode-and-forward nodes should be separated by an equal number of amplify-and-forward relays.

We then analyze benefits of deploying noisy feedback. Our consideration of feedback is motivated by the well known fact that feedback can improve the error exponents, in some scenarios drastically [4]. The error exponents for the point-to-point channel with active noisy feedback for binary signaling were analyzed in [5]. We extend some of these results to the multihop relay network. We first investigate the impact feedback has on the error exponent and thus delay in the single-relay channel. We then extend the analysis to the multihop network.

#### Related Work

For discrete multihop networks with DF relays reliability bounds were analyzed in [6]. Minimizing the delay in Gaussian multihop networks with DF relays was presented in [7]. Error exponents in multihop network with AF relays were analyzed in [8]. Minimizing latency over a microwave networks with DF and AF nodes by considering channel capacity was considered in [9].

The paper is organized as follows. In Section II we define and solve the optimization problem that determines the optimal selection of DF and AF relays. Section III analyzes the impact of feedback. Section IV concludes the paper and discusses future work. In the paper, the proofs of theorems are outlined. Detailed proofs are available in [10].

## II. MULTIHOP NETWORK

We consider a single source-destination multihop wireless network in which data from the source to the destination is transmitted via  $H$  relays (see Fig. 1). Each node is equipped with a single antenna. Following practical constraints, we assume that all transmissions are orthogonal and that each node communicates only with its neighbor, as indicated in Fig. 1. We consider a Gaussian channel where a transmitted signal is corrupted by the additive, white Gaussian noise. The received signal at the node  $k$  is given by

$$y_k = h_{k-1}x_{k-1} + z_k \quad k = 1, \dots, H \quad (1)$$

where the transmit signal at the source is denoted  $x_0$ . The channel gain from a node  $k-1$  to node  $k$  is denoted  $h_{k-1}$ . Noise  $z_k$  has zero mean and variance  $\sigma_k^2$ . Similarly, the received signal at the destination is

$$y_D = h_H x_H + z_D \quad (2)$$

where  $z_D$  is zero mean with variance  $\sigma_D^2$ . To simplify the presentation we assume in this section that  $\sigma_k^2 = \sigma^2$  for all  $k = 1, \dots, H$  and  $k = D$ . The power constraint at node  $k$  is given by

$$E[X_k^2] \leq P_k. \quad (3)$$

The source sends  $B$  bits information intended for the destination node  $D$  using a codeword of length  $n_0$ , by sending a message  $W$  from the message set  $\mathcal{W} = \{1, \dots, 2^B\}$ . The encoding function at the source is given by  $X_0^{n_0} = f(W)$ . A general encoding function at each relay  $k$  at time  $i$  is given by  $X_{k,i} = f_{k,i}(Y_k^{i-1})$ . Each relay in the network performs either

decode-and-forward or amplify-and-forward. The decode-and-forward scheme does not require block Markov encoding [11] because each receiving node receives signal only from one other node. Let  $K \leq H$  denote the number of DF relays. The  $k$ th DF relay performs decoding  $\hat{W}_k = g_k(Y_k^{n_{k-1}})$  where  $\hat{W}_k$  denotes the message estimate at that node and  $n_{k-1}$  denotes codeword length used by  $(k-1)$ th DF node. After decoding, the  $k$ th relay sends a codeword of length  $n_k$ :  $X_k^{n_k} = f_k(\hat{W}_k)$ . On the other hand, a relay  $k$  performing AF, at each time instant  $i$  transmits

$$x_k(i) = \beta_k y_k(i-1) \quad (4)$$

where  $\beta_k$  denotes the amplification gain. From (4) and due to the power constraint at the relay (3),  $\beta_k$  satisfies:

$$\beta_k^2 \leq \frac{P_k}{h_{k-1}^2 P_{k-1} + \sigma^2}. \quad (5)$$

The decoding function at the destination is given by  $\hat{W} = g(Y_D^{n_K})$  where  $n_K$  denotes the codeword length used at the  $K$ th DF relay in the network. The average error probability of the code is  $P_e^{(n_K)} = P[W \neq \hat{W}(Y_D^{n_K})]$ .

Our goal is to minimize the delay in sending messages between the source and the destination while guaranteeing a required level of reliability  $\delta_e$  at the destination, i.e.,

$$P_e^{(n_K)} \leq \delta_e. \quad (6)$$

We consider a problem in which the network is already in place, i.e., the relays are already positioned in the network. Therefore, the number of hops  $H$  and channel gains are given. The considered multihop network is typically a microwave network with a high capacity line-of-sight channel at each hop. The channel variations are much slower compared to a cellular network and thus a transmitter typically has the channel state information. Our goal is to determine a cooperative strategy such that the end-to-end delay is minimized while guaranteeing a required level of reliability (6). In order to minimize the delay, we next review the error exponents and the delay associated with decode-and-forward and amplify-and-forward cooperative schemes.

The error exponent is defined by [3]

$$E_r = - \lim_{n \rightarrow \infty} \sup \frac{1}{n} P_e^{*(n)} \quad (7)$$

where  $P_e^{*(n)}$  denotes the infimum of the error probability over all  $(R, n)$  codes. In a Gaussian point-to-point channel with a received signal-to-noise ratio denoted as SNR, by choosing the Gaussian inputs the error exponent (7) evaluates to

$$E_r \geq \max_{\rho \in [0,1]} \left[ \rho \log \left( 1 + \frac{\text{SNR}}{1+\rho} \right) - \rho R \right]. \quad (8)$$

In order to satisfy a reliability constraint (6), the delay introduced by transmission of a codeword of length  $n_{pp}$  can be calculated from (7) and (8) to be

$$n_{pp} \geq \frac{\rho B - \log \delta_e}{\log \left( 1 + \frac{\text{SNR}}{1+\rho} \right)} \quad (9)$$

where  $\rho \in [0, 1]$  should be chosen so that  $n_{pp}$  is minimized.

Consider a multihop network with  $K$  relays all performing decode-and-forward. Because at each hop the information is decoded, each relay introduces a delay given by (9) and the total delay obtained from (9) is, [7]

$$D_{DF} \geq \sum_{k=1}^{K+1} \frac{\rho_k B - \log \delta_k}{\log(1 + \frac{\text{SNR}_k}{1+\rho_k})} \quad (10)$$

where  $\delta_k$  denotes the required level of reliability at relay  $k$ ,  $\text{SNR}_k$  is received SNR at relay  $k$  and we denoted the destination node as  $K+1$ .

We consider the amplify-and-forward cooperative strategy next. When all  $K$  relays perform amplify-and-forward, it is straightforward to derive from (1), (2), (4) and (5) that the received signal at the destination can be written as

$$y_D(i) = h_{e,K} x_0(i-K) + z_{e,K}(i) \quad (11)$$

where

$$\begin{aligned} h_{e,K} &= h_0 \prod_{i=1}^K \beta_i h_i \\ z_{e,K}(i) &= \sum_{k=1}^K \left( \prod_{j=k}^K \beta_j h_j \right) z_k(i-K+k-1) + z_D(i). \end{aligned} \quad (12)$$

We denote the received SNR in the equivalent channel (11) as  $\gamma(K)$ :

$$\gamma(K) = \text{SNR}_0 \frac{\prod_{i=1}^K (\beta_i h_i)^2}{\sum_{k=1}^K \prod_{i=k}^K (\beta_i h_i)^2 + 1} \quad (13)$$

where  $\text{SNR}_0 = h_0^2 P_0 / \sigma^2$ . We observe that the output (11) is the same as in a point-to-point channel with received SNR  $\gamma(K)$ . From (9), it then follows that the delay introduced by  $K$  AF relays is given by

$$D_{AF} \geq \frac{\rho B - \log \delta_e}{\rho \log(1 + \frac{\gamma(K)}{1+\rho})}. \quad (14)$$

where  $\delta_e$  denotes required end-to-end reliability.

We observe from (10) that each DF relay introduces a delay  $n_{pp}$  given by (9). This is due to the fact that a DF node has to wait to receive the whole codeword prior to forwarding. In contrast, an AF node can forward on symbol-per-symbol, reducing a network of a cascade of AF nodes to a point-to-point channel (11), albeit with reduced SNR. Each AF node reduces the received SNR by amplifying the noise thereby reducing the transmission rate and ultimately increasing the delay. In fact, below a certain value of  $\gamma(K)$ , an AF node will cause a larger delay than a DF node.

To determine the optimum number and positions of AF and DF relays that minimize the end-to-end delay in the considered multihop network, we next define a following optimization problem. Let  $N_{DF}$  denote the number of DF nodes in the network including the source. Let  $K_i \in \{0, \dots, H\}$  denote the number of AF relays in between  $(i-1)$ th and  $i$ th DF relay and let  $p_i$  denote the index (position) of the  $i$ th DF

relay. Then,  $p_1 = 0$ . The delay introduced between  $(i-1)$ th and  $i$ th DF relay is given by (14) for  $K = K_i$ . We formulate the optimization problem as:

$$\begin{aligned} D^* &= \min_{N_{DF}, K_i} \sum_{i=1}^{N_{DF}} \frac{\rho_i B - \log \frac{\delta_e}{N_{DF}}}{\rho_i \log(1 + \frac{\gamma(p_i, K_i)}{1+\rho_i})} \\ \text{s.t. } &\sum_{i=1}^{N_{DF}} K_i + N_{DF} = H + 1 \end{aligned} \quad (15)$$

where

$$\gamma(p_i, K_i) = \frac{\text{SNR}_{p_i} \prod_{j=p_i+1}^{p_i+K_i} (\beta_j h_j)^2}{\sum_{k=p_i+1}^{p_i+K_i} (\prod_{j=k}^{p_i+K_i} \beta_j h_j)^2 + 1} \quad (16)$$

and  $\text{SNR}_{p_i} = h_i^2 P_i / \sigma^2$ . The reliability constraint at each hop is chosen such that, by union bound, the end-to-end reliability constraint is satisfied. The solutions to this problem can efficiently be found by dynamic programming [12]. We present solution for examples of networks later in this section.

The next theorem present a solution to (15) in high SNR. It shows that in the high SNR regime, the minimum delay is obtained when all relays perform AF.

*Theorem 1:* Let  $P_k = s \hat{P}_k$  for all  $k = 0, \dots, H$ . Then

$$\lim_{s \rightarrow \infty} \frac{D_{AF}(s)}{D_{DF}(s)} = \frac{1}{N_{DF}}. \quad (17)$$

*Proof: (Outline)* In high SNR, the SNR after  $K$  stages of AF relays, (13), reduces to

$$\gamma(K) = \left( \sum_{k=0}^K \frac{1}{\text{SNR}_k} \right)^{-1} \quad (18)$$

where we used the notation  $\text{SNR}_k = h_k^2 P_k / \sigma^2$ . The rest of the proof follows from evaluating the delay associated with AF relays (14) and with DF relays (10). ■

*Remark 1:* The intuition for this solution comes from the fact that AF introduces a smaller delay than DF for the price of reduced SNR for each AF hop. In high SNR, however, the SNR loss is negligible and therefore multihop AF is optimal.

We next consider a symmetric network with equal power and channel gains. We consider a subproblem of (15) wherein we optimize  $K_i$ ,  $i = 1, \dots, N_{DF}$  for fixed  $N_{DF}$ . To give an insight to the optimum solution, we relax the constraint that  $K_i$  is an integer. We have the following result.

*Lemma 1:* For a symmetric network with  $N_{DF}$  decode-and-forward nodes, the optimum solution satisfies for all  $i = 1, \dots, N_{DF}$

$$K_i^* = \frac{H}{N_{DF}}. \quad (19)$$

*Proof: (Outline)* The proof follows by forming the Lagrangian for the optimization problem (15) and by deriving the optimality conditions. ■

Therefore the optimal positions of AF and DF nodes are such that all AF nodes are positioned at the equal distance.

For a network with a single relay, Figure 2 shows the delay associated with DF and AF, as a function of channel

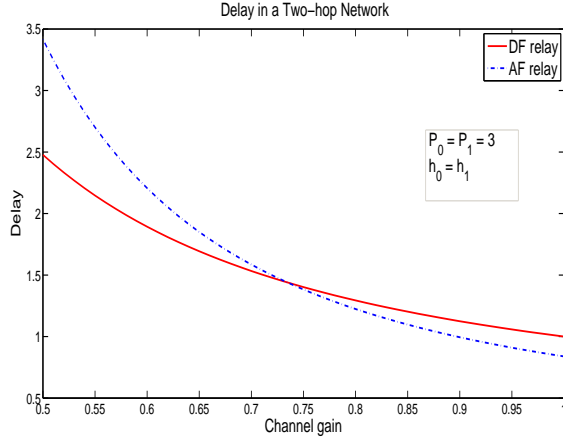


Fig. 2. Delay in a two-hop network.

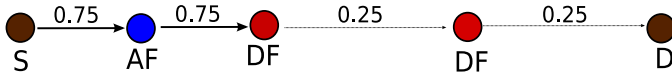


Fig. 3. Optimal selection of AF and DF relays in a 4-hop network. Transmit powers are chosen equal at all nodes. Channel gains are shown for each hop.

gains. Channel gains to and from the relay are chosen equal. We observe that, for smaller values of channel gains, DF at the relay is optimal. After a certain threshold, AF becomes optimal.

Figures 3-5 show a solution of optimization problem (15) for three examples of 4-hop networks. In all three examples the transmit powers at nodes are chosen equal. We observe that the optimal relay selection in Fig. 3 differs from the common practice solution whereby only relay 3 would perform DF. We observe that in the optimum solution, relay 2 chooses to perform DF to compensate for weak links further down in the transmission chain. In contrast, if relay 2 would make decision solely based on its received SNR, it would choose to perform AF. Another example in which the optimal selection differs from the common practice is shown in 4. Fig. 5 shows that for large values of channel gains the optimal solution is AF for all relays, in agreement with Theorem 1.

### III. DELAY IN NETWORKS WITH FEEDBACK

To examine the impact of feedback, we first analyze the network with a single relay. We assume that from each receiving node i.e., the relay and the destination, there is a feedback link (see Fig. 6). The feedback links are orthogonal to the forward links. The forward channel output at the relay is given by (1) and at the destination by (2), for  $H = 1$ . Since there is only one relay, we denote the source and relay inputs as  $x_S$  and  $x_R$  respectively. We use similar notation also for the outputs and the noise at each receiver. The input-output relationship is then given by:

$$\begin{aligned} y_R &= h_S x_S + z_R \\ y_D &= h_R x_R + z_D. \end{aligned} \quad (20)$$

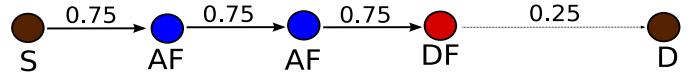


Fig. 4. Optimal selection of AF and DF relays in a 4-hop network. Channel gains are shown for each hop.

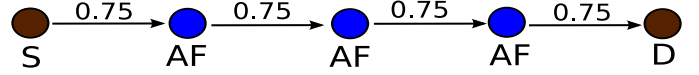


Fig. 5. Optimal selection of AF and DF relays in a 4-hop network. Channel gains are shown for each hop. This network is in high SNR.

As before, the source and the relay satisfy respective power constraints  $E[X_S^2] \leq P_S$  and  $E[X_R^2] \leq P_R$ .

Similarly to the forward channel, the feedback channel is:

$$\begin{aligned} \tilde{y}_R &= \tilde{h}_R \tilde{x}_D + \tilde{z}_R \\ \tilde{y}_S &= \tilde{h}_S \tilde{x}_R + \tilde{z}_S \end{aligned} \quad (21)$$

and the power constraints are given by  $E[\tilde{X}_R^2] \leq \tilde{P}_R$  and  $E[\tilde{X}_D^2] \leq \tilde{P}_D$ . Noises  $\tilde{z}_R$  and  $\tilde{z}_S$  have zero mean and respective variance  $\tilde{\sigma}_R^2$  and  $\tilde{\sigma}_S^2$ .

To analyze the impact of feedback on the delay in this channel, we extend the results of [5] that develops error exponents for the point-to-point channel with active noisy feedback. The analysis in [5] assumes binary signaling, and in the reminder of the paper, we make the same assumption. In particular, we assume that the encoder sends a single bit that takes values 0 or 1 equiprobably. We have the following theorem.

**Theorem 2:** The error exponent achievable in the considered single-relay channel with active noisy feedback is bounded by

$$E_{FB} \geq \frac{2P_S}{\sigma_F^2} + \frac{2\tilde{P}_D}{\sigma_{FB}^2} \quad (22)$$

where

$$\begin{aligned} \sigma_F^2 &= \frac{\sigma_R^2}{h_S^2} + \frac{\sigma_D^2}{(h_S h_R \beta)^2} \\ \sigma_{FB}^2 &= \frac{\tilde{\sigma}_R^2}{\tilde{h}_S^2} + \frac{\tilde{\sigma}_S^2}{(\tilde{h}_S \tilde{h}_R \tilde{\beta})^2} \end{aligned} \quad (23)$$

and the corresponding delay for the reliability level  $\delta_e$  is bounded by

$$n_{FB} \geq \left( \frac{2P}{\sigma_F^2} + \frac{2\tilde{P}}{\sigma_{FB}^2} \right)^{-1} \log \frac{1}{\delta_e}. \quad (24)$$

*Proof:* We consider the amplify-and-forward cooperative strategy at the relay both on the forward and the backward channel. The transmit signal at the relay at time  $i$  in the forward channel is then given by (4) for  $k = R$

$$x_R(i) = \beta y_R(i-1) \quad (25)$$

where  $\beta$  from (5) satisfies:

$$\beta^2 \leq \frac{P_R}{h_S^2 P_S + \sigma_R^2}. \quad (26)$$

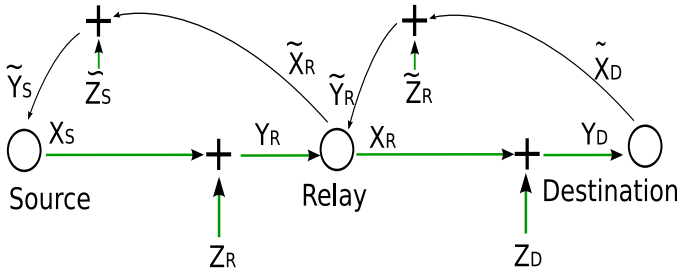


Fig. 6. Single relay channel with per-hop feedback. No direct link is assumed between the source and the destination.

The received signal at the destination evaluates from (11) to be

$$y_D(i) = h_{eq}x_S(i-1) + z_{eq}(i) \quad (27)$$

where

$$h_{eq} = h_R\beta h_S \quad (28)$$

and

$$z_{eq}(i) = h_R\beta z_R(i-1) + z_D(i). \quad (29)$$

Channel (27) is equivalent to the unit-gain channel given by

$$y_D(i) = x_S(i-1) + z_F(i) \quad (30)$$

where  $z_F = z_{eq}/h_{eq}$ . Using (28) and (29) we obtain that the variance of  $z_F$  equals  $\sigma_F^2$  given by (23).

Similarly, the transmit signal at the relay in the reverse channel is given by

$$\tilde{x}_R(i) = \tilde{\beta}\tilde{y}_R(i-1) \quad (31)$$

with amplification gain

$$\tilde{\beta} \leq \frac{\tilde{P}_R}{\tilde{h}_R^2 \tilde{P}_D + \tilde{\sigma}_R^2}. \quad (32)$$

The received feedback signal at the source is given by

$$\begin{aligned} \tilde{y}_S(i) &= \tilde{h}_S \tilde{\beta} \tilde{h}_R \tilde{x}_D(i-1) + \tilde{h}_S \tilde{\beta} \tilde{z}_R(i-1) + \tilde{z}_S(i) \\ &= \tilde{h}_{eq} \tilde{x}_D(i-1) + \tilde{z}_{eq}(i) \end{aligned} \quad (33)$$

where

$$\tilde{h}_{eq} = \tilde{h}_S \tilde{\beta} \tilde{h}_R, \quad (34)$$

and

$$\tilde{z}_{eq}(i) = \tilde{h}_S \tilde{\beta} \tilde{z}_R(i-1) + \tilde{z}_S(i). \quad (35)$$

Again, we can consider the equivalent unit-gain channel

$$\tilde{y}_S(i) = \tilde{x}_D(i-1) + \tilde{z}_{FB}(i) \quad (36)$$

where  $z_{FB} = \tilde{z}_{eq}/\tilde{h}_{eq}$  has the variance  $\sigma_{FB}^2$  given by (23).

The equivalent channel model given by (30) and (36) corresponds to a Gaussian point-to-point channel with a noisy active feedback with noise variances given by (23), analyzed in [5, Sec. VII]. The result in [5, Sec. VII] applies yielding the error exponent given by (22). ■

*Remark 2:* We compare the delay (24) to the delay in the point-to-point channel with the respective noise variances in

the forward and the feedback channel  $\sigma_{D,pp}^2$  and  $\tilde{\sigma}_{S,pp}^2$ , and the respective channel gains  $h_{S,pp}$  and  $\tilde{h}_{D,pp}$ . The error exponent of this channel, as determined in [5, Sec. VII], is given by (22) with  $\sigma_F^2 = \sigma_{D,pp}^2/h_{S,pp}^2$  and  $\sigma_{FB}^2 = \tilde{\sigma}_{S,pp}^2/\tilde{h}_{D,pp}^2$ . By comparing (22) for the relay with the point-to-point channel, we obtain the sufficient conditions under which the delay in the relay channel is smaller:

$$\begin{aligned} \frac{\sigma_R^2}{h_S^2} + \frac{\sigma_D^2}{(h_S h_R \beta)^2} &< \frac{\sigma_{D,pp}^2}{h_{S,pp}^2} \\ \frac{\tilde{\sigma}_R^2}{\tilde{h}_R^2} + \frac{\tilde{\sigma}_S^2}{(\tilde{h}_S \tilde{h}_R \tilde{\beta})^2} &< \frac{\tilde{\sigma}_{S,pp}^2}{\tilde{h}_{D,pp}^2}. \end{aligned} \quad (37)$$

Consider the case when all the noise variances and powers are the same and the channel is in high SNR. Then  $\beta = 1/h_S^2$ ,  $\tilde{\beta} = 1/\tilde{h}_R^2$  and conditions (37) reduce to

$$\begin{aligned} \frac{1}{h_S^2} + \frac{1}{h_R^2} &< \frac{1}{h_{S,pp}^2} \\ \frac{1}{\tilde{h}_R^2} + \frac{1}{\tilde{h}_S^2} &< \frac{1}{\tilde{h}_{D,pp}^2}. \end{aligned} \quad (38)$$

When the above conditions on the channel gains are satisfied, the error exponent and the delay are improved by the help of the relay.

*Remark 3:* We compare the delay (24) to the delay in the considered relay channel when there is no feedback. As before, the relay performs amplify-and-forward and the channel is given by (27). The error probability with binary signaling for this channel is given by

$$P_b = \exp\left(-\frac{h_{eq}^2 P_S}{2\sigma_{eq}^2} n\right) \quad (39)$$

where  $h_{eq}$  is given by (28) and  $\sigma_{eq}^2$  is the variance of  $z_{eq}$  given by (29). The corresponding delay for the reliability level  $\delta_e$  is

$$n \geq \frac{2\sigma_{eq}^2}{h_{eq}^2 P_S} \log \frac{1}{\delta_e}. \quad (40)$$

Delay (24) and (40) can be easily compared for case that  $P = \tilde{P}$  and all noises have the same variance and  $\sigma_F \geq \sigma_{FB}$  in (23). In this case, the delay with feedback is always smaller than the delay in the no-feedback channel.

We can now extend the result of Thm. 2 to multihop network with active noisy feedback between every transmitter/receiver pair as shown in Fig. 7.

*Theorem 3:* The error exponent achievable in the multihop relay network with  $K$  relays and with active noisy feedback is bounded by

$$E_{FB} \geq \frac{2P_S}{\sigma_F^2} + \frac{2P_{FB}}{\sigma_{FB}^2} \quad (41)$$

where

$$\begin{aligned} \sigma_F^2 &= \frac{\sum_{k=1}^H \prod_{i=k}^H (\beta_i h_i)^2 \sigma_k^2 + \sigma_D^2}{h_0^2 \prod_{i=k}^H (\beta_i h_i)^2} \\ \sigma_{FB}^2 &= \frac{\sum_{k=1}^H \prod_{j=k}^H (\tilde{\beta}_j \tilde{h}_{j-1})^2 \tilde{\sigma}_k^2 + \tilde{\sigma}_S^2}{\tilde{h}_0^2 \prod_{k=1}^H (\tilde{\beta}_k \tilde{h}_k)^2}. \end{aligned} \quad (42)$$

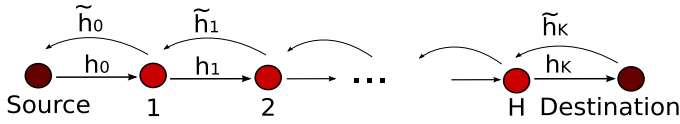


Fig. 7. Multihop network with feedback.

*Proof:* The proof follows the same steps as the proof of Theorem 2. ■

#### Discussion

The above results show the improvement in terms of the required codelength  $n$  over the no-feedback case. However, the analysis presented in the previous section does not capture the propagation delay. In high-frequency trading applications where data is sent over multiple hops and hundreds of kilometers, the propagation delay is a dominant factor. Using feedback at every link is not appropriate as it would significantly increase the propagation delay. Instead, the presented analysis points that a feedback could potentially reduce the delay when used sporadically, on certain hops.

#### IV. SUMMARY AND FUTURE WORK

We have presented an information-theoretic approach that, based on error exponents, optimally determines a cooperative strategy between DF and AF for every relay. We showed that in high SNR the optimum transmission scheme is for all relays to perform AF. We have then derived the error exponents in the considered multihop network in the presence of noisy, active feedback.

Our results demonstrate that the choice of the cooperative scheme cannot be made solely based on the received SNR at that relay, as is a common practice. Instead, this choice depends on the channel conditions in the whole network.

In practice, in addition to the average power constraint, the peak power constraint needs to be satisfied. This motivates extending the presented analysis under the peak power constraints. Achievable error exponents under the peak power constraints for the point-to-point channel were derived in [13]. Furthermore, the considered problem can be extended to include slow fading analysis. In addition to decode-and-forward and amplify-and-forward schemes analyzed in this paper, these results can be extended also to compress-and-forward [11]. Another open problem is investigating whether a form of feedback, possibly used only at certain time instants and at certain nodes, could improve the end-to-end delay.

#### V. ACKNOWLEDGEMENT

The author would like to thank Andrea Montanari for inspiring and insightful discussions.

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